

3.4

Polynomial DNA

Key Characteristics of Polynomial Functions

LEARNING GOALS

In this lesson, you will:

- Interpret polynomial key characteristics in the context of a problem situation.
- Generalize the key characteristics of polynomials.
- Sketch the graph of any polynomial given certain key characteristics.

KEY TERMS

- absolute maximum
- absolute minimum
- extrema

Children typically resemble their parents because of the inheritance of genes from parent to offspring. Scientists know of over 200 hereditary traits that are transmitted across generations of families. The genes that carry these traits are in specific strands of DNA. You can witness these traits by crossing your hands. Is your left thumb over your right thumb? If it is, you have the dominant trait. People with the recessive trait will cross their right thumb over their left thumb. Try it the opposite way, it feels awkward doesn't it?

Did you ever work with Punnett squares in biology to determine the probability of an offspring having a particular characteristic like blue eyes versus brown eyes or eyelash length? Being able to roll your tongue is actually a dominant genetic feature. Some other dominant genetic human traits are non-cleft chins, widow's peaks, broad eyebrows, freckles, dimples, and unattached ear lobes to name a few. When you look at the specific genotype of a species you can determine or predict what the offspring may look like.

The same thing is true for polynomials! If you know certain characteristics about the polynomial, you can predict what the graph will look like, as well as other key characteristics.

PROBLEM 1 Math World vs. Real World



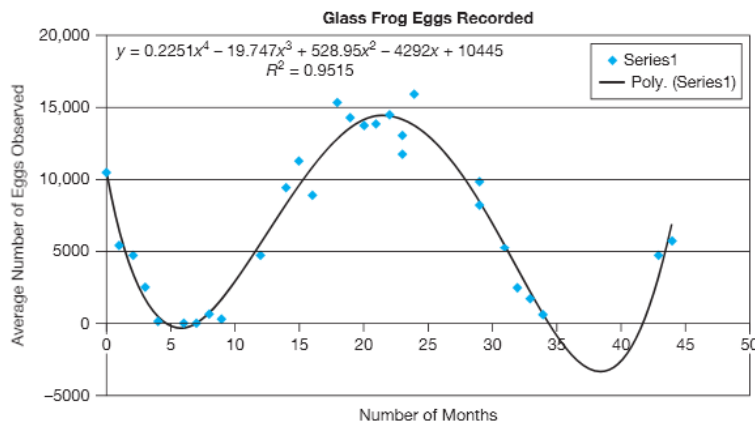
The data shown represents the population of a rare, endangered species of frog called the glass frog. In order to better understand the glass frog's fertilization habits, scientists performed a study and recorded the average number of frog eggs over the span of 44 months.

3

Month of Study	Average Number of Glass Frog Eggs
0	10,534
1	5500
2	5033
3	2600
4	239.4
6	137.3
7	108.4
8	667.1
9	387.4
12	4813.1
14	9539.5
15	11,318.6
16	8953.3
18	15,402.5

Month of Study	Average Number of Glass Frog Eggs
19	14,330.5
20	13,845.1
21	13,893.1
22	14,546.3
23	11,815.8
23	13,086.2
24	15,966.9
29	9904.4
29	8257.3
31	5297.5
32	2494.1
33	1805.4
34	665
43	4813

The data has been plotted for you and a quartic regression was used to generate the polynomial function to best represent the data. The quartic regression option calculates the best-fit equation of the form $y = ax^4 + bx^3 + cx^2 + dx + e$.



© Carnegie Learning



1. Consider the graph and equation to answer each question.
 - a. What is the domain and range of the study?
 - b. Explain what the domain and range represent in the context of this problem.
 - c. What is the domain and range of the function?
 - d. At what month in the study were the most frog eggs observed? How many eggs were recorded?
 - e. At what month in the study were the least frog eggs observed? How many eggs were recorded?
 - f. If the study lasted for 50 months, how many frog eggs would there be according to the function?
 - g. If the study lasted forever, how many eggs would there be according to the function?
 - h. How many frog eggs appeared between months 35 and 40?
 - i. At what month(s) of the study were there approximately 4800 glass frog eggs observed?
2. Use a graphing calculator to determine the x -intercepts of the function. What do the x -intercepts mean in the context of this problem situation?



3. State the end behavior of the function. Does this make sense in the context of this problem scenario? Explain your reasoning.

4. How many frog eggs were observed at the beginning of the study? Explain the mathematical meaning of your answer.

3

5. Describe the interval when the frog's egg population is:
 - a. increasing.



- b. decreasing.

PROBLEM 2 A Polynomial is Born



So far in this chapter, you have learned a great deal about polynomial functions. You have learned about minimums, maximums, zeros, end behavior, and the general shapes of their graphs. Now, you will combine all that information to generalize the key characteristics for any degree polynomial.

Recall the definition of a **relative maximum** is the highest point in a particular section of a function's graph, and a **relative minimum** is the lowest point in a particular section of the graph. Similarly, the **absolute maximum** is the highest point in the entire graph, and the **absolute minimum** is the lowest point in the entire graph. The set of absolute maximums, absolute minimums, relative maximums, and relative minimums may also be referred to as **extrema**. The extrema are also called extreme points and extremum.



1. Consider the graph that represents the average number of glass frog eggs in Problem 1.
 - a. State all relative maximums and minimums.

 - b. State all absolute maximums and minimums.



c. Do the absolute minimums and/or maximums make sense in the context of this problem situation? Explain your reasoning.

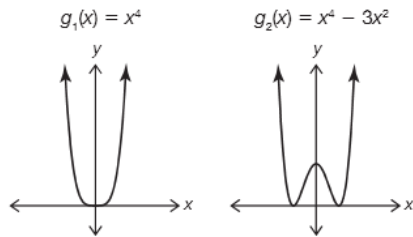


2. Determine the number of extrema in each polynomial.



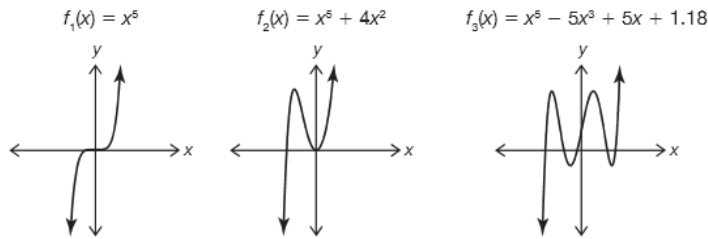
3

4th Degree Polynomials



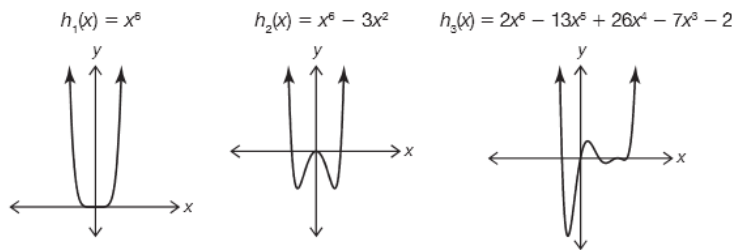
Number of Extrema

5th Degree Polynomials



Number of Extrema

6th Degree Polynomials



Number of Extrema

3. List any observations you notice about the possible number of extrema and the degree of the polynomial.

4. List the possible number of extrema for the each polynomial.

a. 9th degree polynomial

b. 18th degree polynomial

c. n th degree odd polynomial

d. n th degree even polynomial

Use the knowledge you gained about 4th, 5th, and 6th degree polynomials to answer these questions.



3

5. Choose the appropriate word from the box to complete each statement. Justify your answer with a sketch or explanation.

always	sometimes	never
--------	-----------	-------

a. An odd degree function will _____ have absolute extrema.

b. An even degree function will _____ have relative extrema.

c. An even degree function will _____ have 3 or more relative extrema.

d. An even degree function will _____ have absolute extrema.

e. An odd degree function will _____ have relative extrema.

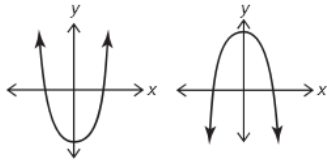
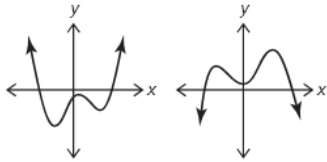
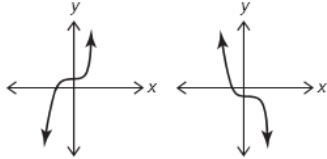
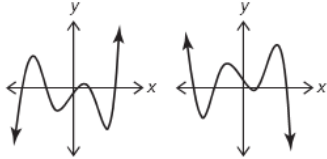


f. An odd degree function will _____ one have relative extrema.

3



6. Analyze the graphs shown.

Even Degree Power Functions	Even Degree Polynomial Functions
	
Odd Degree Power Functions	Odd Degree Polynomial Functions
	

a. State the similarities and differences you notice between the power functions and the polynomial functions.

b. What conclusions can you make about the end behavior of all even degree polynomial functions?

- c. What conclusions can you make about the end behavior of all odd degree polynomial functions?
- d. What conclusions can you make about the domain and range of all even degree polynomial functions?

3

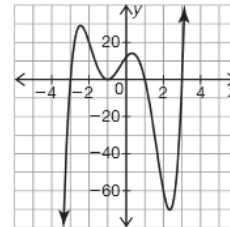


- e. What conclusions can you make about the domain and range of all odd degree polynomial functions?



7. Consider the graph shown.

- a. Is the a -value of this function positive or negative?



- b. Is the degree of this function even or odd?

- c. Can this function be a cubic function? Explain why or why not.

- d. State the domain of this function.

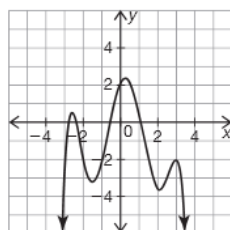
- e. State the range of this function.

- f. Determine the number of relative extrema in this graph.

- g. Determine the number of absolute extrema in this graph.

- h. State the intervals where the graph is increasing.

8. Consider the graph shown.



a. Is the a -value of this function positive or negative?

b. Is the degree of this function even or odd?

c. Can this function be a 6th degree polynomial function? Explain why or why not.

d. State the domain of this function.

3

e. State the range of this function.

f. Determine the number of relative extrema in this graph.

g. Determine the number of absolute extrema in this graph.



h. State the intervals where the graph is decreasing.

Wow! You know a lot about graphs of polynomials.





9. Complete the table on the next page to represent the graphs of various polynomials.
- Sketch the basic shape on each set of axes, given the number of zeros. If you cannot sketch the basic shape, explain why.
 - Compare your graphs with a partner. State the similarities and differences.

After you complete the table, answer parts (b) through (e).



3

- What do you notice about the maximum number of x -intercepts and the degree of the function?
- Use your graphs to determine the greatest number of extrema (absolute and relative) in each degree polynomial.

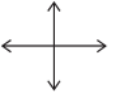
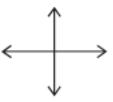
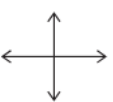
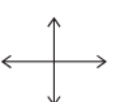

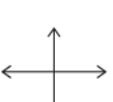
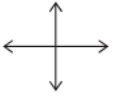
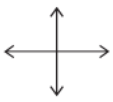
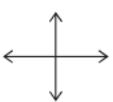
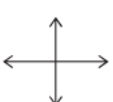
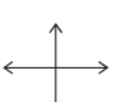
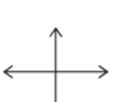
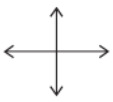
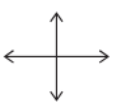
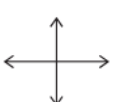
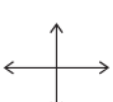
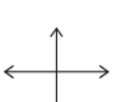
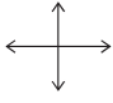
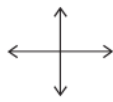
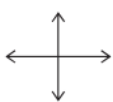
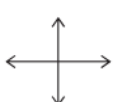


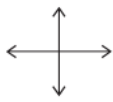
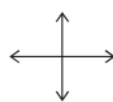
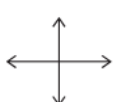
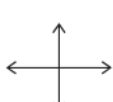
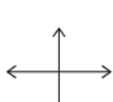
Type of Polynomial Function	Number of Extrema
Linear	
Quadratic	
Cubic	
Quartic	
Quintic	



- What do you notice about the number of extrema and the degree of a polynomial? Write a statement to generalize the possible number of extrema in any degree polynomial function.

© Carnegie Learning

© Carnegie Learning

	No Zeros	1 Zero	Exactly 2 Zeros	Exactly 3 Zeros	Exactly 4 Zeros	Exactly 5 Zeros
Linear						
Quadratic						
Cubic						
Quartic						
Quintic						

3

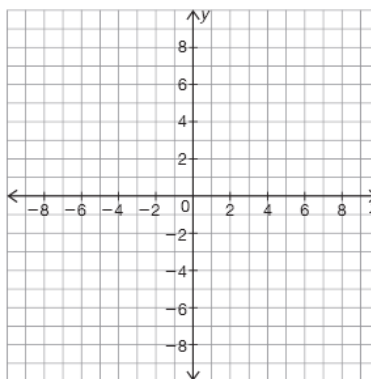
PROBLEM 3 Who Am I?



1. Use the coordinate plane to sketch a graph with the characteristics given. If the graph is not possible to sketch, explain why.

a. Characteristics:

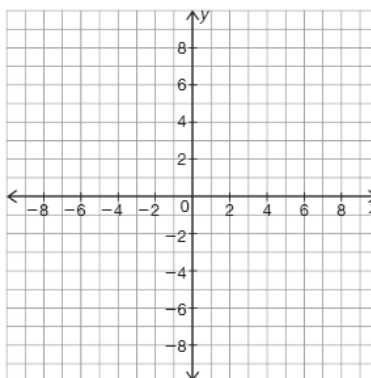
- degree 4
- starts in quadrant III
- ends in quadrant IV
- relative maximum at $x = -4$
- absolute maximum at $x = 3$



3

b. Characteristics:

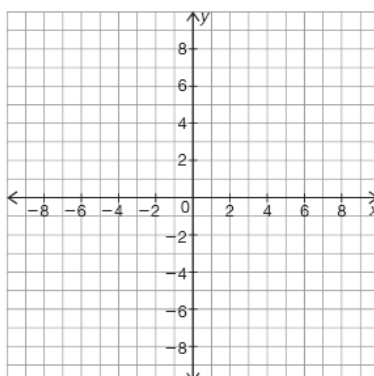
- always increasing
- y-intercept at 5
- x-intercept at -1.7



© Carnegie Learning

c. Characteristics:

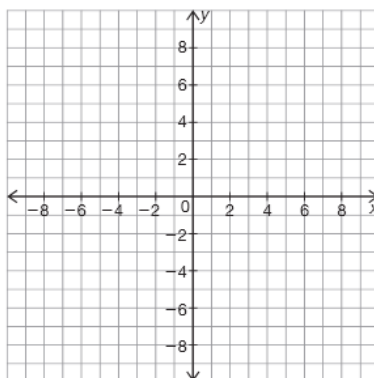
- odd degree
- increases to $x = -3$, then decreases to $x = 3$, then increases
- absolute maximum at $y = 4$



3

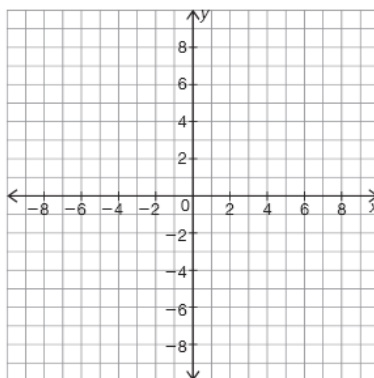
d. Characteristics:

- as $x \rightarrow \infty, f(x) \rightarrow \infty$
as $x \rightarrow -\infty, f(x) \rightarrow \infty$
- 4 x -intercepts
- relative maximum at $y = 3$



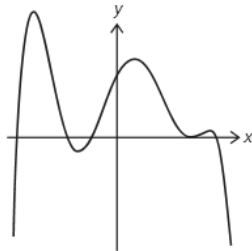
e. Characteristics:

- x -intercepts at $-2, 2$ and 5
- negative a value
- degree 2



2. Analyze each graph. Circle the function(s) which could model the graph. Describe your reasoning to either eliminate or choose each function.

a. $f_1(x) = -3x^5 - 2x^2 + 4x + 7$



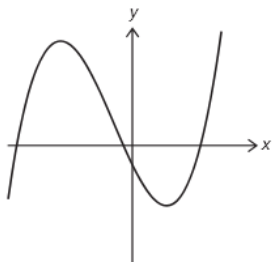
$f_2(x) = -(x + 2)(x + 1.5)(x + 0.5)(x - 2.5)^2(x - 3)$

$f_3(x) = -3x^4 - 2x^2 + 4x + 7$

3



b. $f_1(x) = 0.5(x + 7)(x + 1)(x - 5) - 3$



$f_2(x) = -2(x + 7)(x + 1)(x - 5) - 3$

$f_3(x) = 2(x + 7)(x + 1)(x - 5)(x - 3)$

© Carnegie Learning

Talk the Talk



Complete each table to summarize the key characteristics for quartics and quintics.

The cubics table has been done for you.

Cubics	
All possible end behavior	$\text{As } x \rightarrow \infty, f(x) \rightarrow \infty.$ $\text{As } x \rightarrow -\infty, f(x) \rightarrow -\infty.$ $\text{As } x \rightarrow \infty, f(x) \rightarrow -\infty.$ $\text{As } x \rightarrow -\infty, f(x) \rightarrow \infty.$
Possible number of x-intercept(s)	3, 2, or 1
Possible number of y-intercept(s)	1
Possible intervals of increase and decrease	<ul style="list-style-type: none"> • Always increasing • Always decreasing • Increasing, decreasing, increasing • Decreasing, increasing, decreasing
Number of possible relative extrema	2 or none
Number of possible absolute extrema	None

3

3

Quartics	
All possible end behavior	
Possible number of x -intercept(s)	
Possible number of y -intercept(s)	
Possible intervals of increase and decrease	
Number of possible relative extrema	
Number of possible absolute extrema	

Quintics	
All possible end behavior	
Possible number of x -intercept(s)	
Possible number of y -intercept(s)	
Possible intervals of increase and decrease	
Number of possible relative extrema	
Number of possible absolute extrema	



Be prepared to share your solutions and methods.